Journal of civil engineering and transport **transEngin**

2025, Volume **7** Number **1** **ISSN 2658-1698, e-ISSN 2658-2120**

DOI: 10.24136/tren.2025.001

ESTIMATION OF UNCERTAINTY OF NATURAL GAS VOLUME LOSS IN THE ASPECT OF DAMAGE TO PIPELINE TRANSPORT INFRASTRUCTURE

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Reviewed positively: 22.01.2025

Information about quoting an article:
Matiko F., Dzhyhyrei V., Matiko H. (2025). Estimation of uncertainty of natural gas volume loss in the aspect of damage to pipeline transport infrastructure. *Journal of civil engineering and transport*. 7(1), 9-18, ISSN 2658-1698, e-ISSN 2658-2120, DOI: 10.24136/tren.2025.001

Abstract – The analytical dependencies for estimating the uncertainty of the volume of gas lost through damage to the gas pipeline infrastructure are developed in this article. The stage of pseudo-stationary gas leakage is considered and equations for estimating the uncertainty of the gas pressure and temperature at the damage point of the gas pipeline are presented. They are based on the solution of the mathematical model of stationary gas movement in the gas pipeline. An equation for the uncertainty of the gas flow rate through the holes in the damaged gas pipeline is obtained based on the analysis of the gas flow equation as a functional dependence on the gas parameters and characteristics of the leakage. An uncertainty budget is formed for gas pressure and temperature at the damage point and gas flow rate through the damage holes. Formulas for the impact coefficients of the uncertainties of the input parameters on the combined standard uncertainty of these parameters are also developed. An example of the application of the developed equations for estimating the uncertainty of the gas volume lost through damage to the gas transportation network is presented.

Key words – pipeline transport infrastructure, gas pipeline damage, gas losses, mathematical model, gas volume uncertainty

JEL Classification – L95, R41

INTRODUCTION

Natural gas is a convenient fuel used by millions of consumers and a raw material for industry. Pipeline networks are the most economical type of gas transportation, from extraction and production to areas of use and processing. The main advantages of pipeline gas transportation are the continuity of the transportation process and its high efficiency, the possibility of intermediate gas storage and full automation of the transportation process, which ensures its reliability. However, during the gas pipeline operation, damages may occur because of the impact of technogenic and natural factors, which leads to gas loss and disruption of the transportation process, which, as a result, causes additional losses to gas transportation and gas distribution organizations.

The negative impact of natural gas leaks on the environment is worth mentioning. Methane, the main component of natural gas, has a strong greenhouse effect [1-2]. Methane emissions from natural gas transportation systems are a significant source of air pollution, which harms the environment.

In the conditions of the ongoing war in Ukraine, the pipeline transport infrastructure operates under constant risks and is often the object of armed attacks. The destruction of pipeline transport infrastructure is a severe blow to the energy sector and is accompanied by the loss of large gas volumes.

In each case of damage to pipeline infrastructure, it is crucial to eliminate the damage as soon as possible, minimize gas losses, determine the volume of gas lost during the damage, and assess the uncertainty of the lost volume of gas.

In this article, the authors present the results of developing analytical dependencies for estimating the uncertainty of the volume of gas lost through damage to a gas pipeline. These dependencies were obtained based on the analysis and solution of a mathematical model of the steady-state gas flow in a gas pipeline.

1. ANALYSIS OF RESEARCH ON ESTIMATION OF UNCERTAINTY OF GAS VOLUME LOST DURING DAMAGE TO GAS TRANSPORTATION INFRASTRUCTURE

The flow rate and volume of natural gas are often measured by indirect measurement methods, and their values are calculated using the measurement method equation. The flow rate (volume) is a functional dependence on the measured natural gas parameters. In this case, the uncertainty of the flow rate or volume of gas is calculated according to equations obtained based on a well-known approach to estimating the uncertainty of a function of several arguments [3-4].

The standard [4] provides guidelines for estimating uncertainties in measuring a fluid's flow rate or volume, including natural gas. The standard describes methods for calculating the uncertainties of fluid flow measurement, including identifying all potential sources of uncertainty, classifying them and quantifying their impact on the measurement results. The standard [4] is used for various fluid flow measurement methods, including pressure differential devices, pitot tubes and volumetric flowmeters. In particular, using the principles and methods set out in the standard [4], a technique for estimating the uncertainty of flow rate measured by the pressure differential method has been developed. It is used in the standard [5].

To estimate uncertainty in practical problems of flow rate and volume measurement problems, researchers often use the approaches outlined in [1-2] or optimization methods, particularly the Monte Carlo method [6]. Optimization methods are used to confirm the correctness of analytical equations for estimating uncertainty. However, analytical equations are the main ones as a practical tool for compiling a budget for uncertainty in flow rate and volume measurement and for estimating the combined standard uncertainty of the measured flow rate and volume [6-8].

In the article [9], two approaches to the estimation of the uncertainty of flow rate measurement using a standard orifice plate are considered: analytical equations and the Monte Carlo method. The authors gave examples of uncertainty estimation for specific flowmeters and presented a detailed methodology for estimating the uncertainty based on the approaches defined in [3-4].

Thus, well-known sources present general approaches and methods for estimating the uncertainty of the result of flow rate and volume measurement and methods for estimating the uncertainty of flow rate for instrumentation systems based on flowmeters and gas meters. However, no methods are proposed for estimating the uncertainty of the volume of natural gas lost through damage to the gas pipeline. Therefore, the development of analytical dependencies for estimating the uncertainty of the volume of lost gas and the methods for applying these dependencies are important tasks for ensuring the balancing of gas transportation networks.

2. MATHEMATICAL MODELS FOR CALCULATING THE VOLUME OF LOST GAS

When the gas transportation infrastructure is damaged, balance methods are often used to determine the volume of lost gas based on measuring the gas flow rate on all branches from the damaged section of the gas pipeline and applying the balance equation to determine the volume of lost gas. However, in many cases, damaged sections of the gas pipeline network are not equipped with gas flowmeters or are partially equipped, so it is impossible to apply balance methods. Then, the volume of gas losses can be determined based on mathematical models of natural gas movement in the gas pipeline and a model of its leakage through holes in the gas pipeline. To determine the gas parameters at any point along the gas pipeline, the authors proposed using an improved mathematical model of natural gas movement in gas pipelines [10], which has the form:

$$\begin{cases} \dfrac{dp}{dx} = -\left[\dfrac{M \cdot g \cdot \Delta y \cdot p^2}{z \cdot R \cdot T \cdot L} + \dfrac{8 \cdot \lambda \cdot q_m^2 \cdot z \cdot R \cdot T}{M \cdot \pi^2 \cdot D^5}\right] \Bigg/ \left(p - \dfrac{16 \cdot q_m^2 \cdot z \cdot R \cdot T}{p \cdot M \cdot \pi^2 \cdot D^4}\right); \\ z = f\left(p, T, w_{N2}, w_{CO2}, \rho_{bc}\right); \\ \dfrac{dT}{dx} = -\left[\dfrac{k_t \cdot \pi \cdot D_{outer}}{q_m \cdot c_p}\left(T - T_{am}\right) - \left(D_i + \dfrac{16 \cdot q_m^2 \cdot z^2 \cdot R^2 \cdot T^2}{c_p \cdot M^2 \cdot \pi^2 \cdot D^2 \cdot p^3}\right)\dfrac{dp}{dx} + \dfrac{g \cdot \Delta y}{c_p \cdot L}\right], \end{cases} \tag{1}$$

where:
- p - is the pressure of natural gas in the gas pipeline;
- T - is the absolute temperature of natural gas;
- T_{am} - is the absolute ambient temperature;
- q_m - is the mass flow rate of natural gas;
- g - is the acceleration of gravity;
- Δy - is the difference between final y_2 and initial y_1 heights of gas pipeline placement;
- L - is the length of gas pipeline;
- D - is the internal diameter of gas pipeline;
- D_{outer} - is the external diameter of gas pipeline;
- R - is the gas constant;
- z - is the compressibility factor of natural gas;
- ρ_{bc} - is the density of natural gas under base conditions;
- M - is the molar mass of natural gas;
- w_{N2} - is the molar fraction of nitrogen;
- w_{CO2} - is the molar fraction of carbon dioxide;
- λ - is the hydraulic resistance coefficient;
- c_p - is the isobaric heat capacity of natural gas;
- k_t - is the heat transfer coefficient from gas to soil;
- D_i - is the Joule-Thomson coefficient.

A change in pressure and temperature profiles occurs along the gas pipeline during its damage [11-12]. Mathematical model (1) makes it possible to calculate the pressure and temperature profile along the gas pipeline and calculate the gas parameters at the pipeline damage point.

To calculate the gas flow rate caused by damage to a gas pipeline, the authors developed a dependence based on the gas velocity formula during its adiabatic leakage through the nozzle (Saint-Venant-Wantzel formula) considering the conditions of the subcritical ($p_b/p > 0.54$) and critical ($p_b/p \leq 0.54$) flow of gas leakage into the air [10]:

where:
- Q_x - is the gas flowrate through the damage to the pipeline at base conditions, m^3/sec;
- C_f - is the flow rate coefficient;
- F_{hole} - is the area of the gas outflow hole, m^2;
- p - is the absolute gas pressure, Pa;
- p_{bar} - is the barometric pressure, Pa;
- K - is the gas compressibility coefficient.

The authors propose to calculate the flow rate coefficient C_f using the equation developed for the gas pressure range in the pipeline from 0.1 to 1.2 MPa [10]:

The methodological error of flow rate coefficient calculation according to equation (3) equals 1.7 %, considering the error of the reference data based on which this equation was developed [10]. For a pressure greater than 1.2 MPa, the flow coefficient is proposed to be calculated by the formula (35) from [13].

Then, the amount of gas lost because of damage is found by integrating the flow rate over the time of gas leakage.

$$Q_x = \begin{cases} 0.1564 \cdot C_f \cdot F_{hole} \cdot p \cdot \sqrt{\dfrac{1}{\rho_{bc} TK} \cdot \left[\left(\dfrac{p_{bar}}{p} \right)^{1.53} - \left(\dfrac{p_{bar}}{p} \right)^{1.77} \right]}, & \text{for } \left(\dfrac{p_{bar}}{p} \right) > 0.54; \\ 0.0359 \cdot C_f \cdot F_{hole} \cdot \dfrac{p}{\sqrt{T \cdot K \cdot \rho_{bc}}}, & \text{for } \left(\dfrac{p_{bar}}{p} \right) \leq 0.54, \end{cases} \quad (2)$$

$$C_f = 0.588 \left(\frac{p_{bar}}{p} \right)^3 - 0.983 \left(\frac{p_{bar}}{p} \right)^2 + 0.163 \left(\frac{p_{bar}}{p} \right) + 0.843. \quad (3)$$

3. Development of analytical dependencies for estimation of the uncertainty of the lost gas volume

The mathematical model (1) is a system of nonlinear differential equations. Therefore, it is solved numerically to obtain the distributions of gas pressure and temperature along the gas pipeline. For some simplified cases, for example, for a horizontal section of the gas pipeline and under the condition of applying some assumptions, the system of equations (1) can have an analytical solution. This analytical solution has the form:

$$p_x = \sqrt{p_1^2 + 2W_u \cdot q_{bc}^2 \cdot \rho_{bc}^2 \cdot z \cdot T \cdot x} \tag{4}$$

$$W_u = -\frac{\lambda \cdot R}{2D \cdot M \cdot F^2} \tag{5}$$

$$T_x = T_{am} + \left(T_1 - T_{am}\right) \cdot e^{-\frac{k_t \cdot \pi \cdot D_{outer}}{q_{bc} \cdot \rho_{bc} \cdot c_p} \cdot x} = T_{am} + \left(T_1 - T_{am}\right) \cdot e^{-a \cdot x} \tag{6}$$

where:
- p_x - is the pressure of natural gas at the damage point;
- p_1 - is the pressure of natural gas upstream of the damage point;
- T_x - is the temperature of natural gas at the damage point;
- T_1 - is the temperature of natural gas upstream of the damage point;
- q_{cb} - is the volumetric gas flow rate reduced to standard conditions;
- x - is the length of the gas pipeline section from the station to the damage;
- F - is the cross-sectional area of the gas pipeline.

In this article, the analysis of the uncertainty of the pressure p_x and the temperature T_x of gas at the point of damage to the gas pipeline is performed based on a simplified solution of system (1), i.e., based on equations (4), (6).

To obtain the uncertainty equation of the result of the calculation of the gas pressure at the pipeline damage point, we considered equation (4) as a functional dependence.

$$p_x = f(p_1, q_{dc}, \rho_{dc}, z, T, x) \tag{7}$$

The procedure for estimating the relative combined standard uncertainty, defined by equation [3-4], was applied to dependence (7):

$$u_c'(Y) = \sqrt{\sum_{i=1}^{N} \vartheta_{x_i}^2 u'(x_i)^2} \tag{8}$$

where:
- $u'(x_i)$ - is the uncertainty of parameter x_i, $x_i = [p_1, q_{bc}, \rho_{bc}, z, T, x]$;
- ϑ_{xi} - is the impact coefficient of the uncertainty of parameter;
- x_i - on the uncertainty of gas pressure, which should be determined by the formula

$$\vartheta_{x_i} = \frac{\partial f}{\partial x_i} \cdot \frac{x_i}{Y}, \text{ де } Y = f(x_i) \tag{9}$$

In this case, we assume that the parameters of the function Y are uncorrelated quantities.

Applying (8), we obtain the equation of the relative standard uncertainty of the natural gas pressure at the damage point of the gas pipeline:

$$u_p' = \sqrt{\vartheta_{p_1}^2 u_{p_1}'^2 + \vartheta_{q_{bc}}^2 u_{q_{bc}}'^2 + \vartheta_{\rho_{bc}}^2 u_{\rho_{bc}}'^2 + \vartheta_z^2 u_z'^2 + \vartheta_T^2 u_T'^2 + \vartheta_x^2 u_x'^2} \tag{10}$$

By applying (9) to the functional dependence (7), the formulas for the impact coefficients of the uncertainty components included in equation (4) were obtained and presented in Table 1.

Table 1. Impact coefficients of uncertainty components for equation [10]

№	Impact coefficient of the uncertainty component	
	Symbol	Formula
1	ϑ_{p_1}	p_1^2/p_x^2
2	$\vartheta_{q_{bc}}$, $\vartheta_{\rho_{bc}}$	$1-p_1^2/p_x^2$
3	ϑ_z , ϑ_T , ϑ_x	$\frac{p_x^2-p_1^2}{2p_x^2}$

Similarly, by applying the procedure for estimating the relative combined standard uncertainty to the functional dependence (6), the equation for the uncertainty of the natural gas temperature at the damage point of the gas pipeline was obtained:

$$u'_T=\sqrt{\vartheta_{T_{am}}^2 u'^2_{T_{am}}+\vartheta_{T_1}^2 u'^2_{T_1}+\vartheta_{q_{bc}}^2 u'^2_{q_{bc}}+\vartheta_{\rho_{bc}}^2 u'^2_{\rho_{bc}}+\vartheta_x^2 u'^2_x} \tag{11}$$

where:
- u'_{xi} - is the relative standard uncertainty of the measured value of a parameter included in the set $x_i = [T_1, T_{am}, q_{bc}, \rho_{bc}, x]$.

The formulas for the impact coefficients of the uncertainty components included in equation (11) were obtained by applying formula (9) to the functional dependence (6). Table 2 presents these formulas.

Table 2. Impact coefficients of uncertainty components for equation [11]

№	Impact coefficient of the uncertainty component	
	Symbol	Formula
1	$\vartheta_{T_{am}}$	$\left(1-e^{-a\cdot x}\right)\cdot\frac{T_{am}}{T_x}$
2	ϑ_{T_1}	$e^{-a\cdot x}\cdot\frac{T_1}{T_x}$
3	ϑ_x	$-\frac{a\cdot x}{T_x}\cdot\left(T_1-T_{am}\right)e^{-a\cdot x}$
4	$\vartheta_{q_{bc}}$	$\left(T_1-T_{am}\right)\cdot e^{-a\cdot x}\cdot\left(\frac{k_t\cdot\pi\cdot D_{outer}\cdot x}{\rho_{bc}\cdot c_p}\right)\frac{1}{q_{bc}\cdot T_x}$
5	$\vartheta_{\rho_{bc}}$	$\left(T_1-T_{am}\right)\cdot e^{-a\cdot x}\cdot\left(\frac{k_t\cdot\pi\cdot D_{outer}\cdot x}{q_{bc}\cdot c_p}\right)\frac{1}{\rho_{bc}\cdot T_x}$

The developed equations (10) and (11) provide the possibility of estimating the uncertainty of the gas pressure and temperature at the pipeline damage point. The uncertainty of the gas flow rate through the damages can be calculated based on the uncertainties of these parameters.

By applying the procedure for estimating the relative combined standard uncertainty to the functional dependence (2), the equation of the uncertainty of the natural gas flow rate through damage to the gas pipeline was obtained:

$$u'_Q = \sqrt{\vartheta_{C_f}^2 u'^2_{C_f} + \vartheta_{F_{hole}}^2 u'^2_{F_{hole}} + \vartheta_p^2 u'^2_p + \vartheta_{p_{bar}}^2 u'^2_{p_{bar}} + \vartheta_\chi^2 u'^2_\chi + \vartheta_T^2 u'^2_T + \vartheta_{\rho_{bc}}^2 u'^2_{\rho_{bc}} + \vartheta_K^2 u'^2_K} \quad (12)$$

where:

- u'_{xi} - is the relative standard uncertainty of the measured value of a parameter included in the set x_i = [C_f, F_{hole}, p, p_{bar}, χ, T, ρ_{bc}, K].

Applying formula (9) to functional dependence (6), we obtained formulas for the impact coefficients of the uncertainty components included in equation (12) and presented them in Table 3.

4. EXAMPLE OF ESTIMATING THE UNCERTAINTY OF THE LOST GAS VOLUME

Let us calculate the relative standard uncertainty of the natural gas flow rate caused by damage to the gas pipeline. Since (12) depends on the uncertainty of the pressure and temperature of natural gas at the damage point of the gas pipeline, it is necessary to calculate the gas pressure and temperature at the damage point under the condition that the gas pipeline has completely ruptured. It is necessary to calculate the values of the impact coefficients of the uncertainty components, the formulas for which are given in Tables 1 and 2. We calculated the uncertainties for a gas pipeline section with the characteristics presented in Table 4.

Table 3. Impact coefficients of uncertainty components for equation [12]

№	Impact coefficient of the uncertainty component	
	Symbol	Formula
1	ϑ_{C_f}	1.0
2	$\vartheta_{F_{hole}}$	1.0
3	ϑ_p	$\begin{cases} \frac{p\left(0.47p^{-0.53}\cdot p_{bar}^{1.53} - 0.23p^{-0.77}\cdot p_{bar}^{1.77}\right)}{2\left(p^{0.47}\cdot p_{bar}^{1.53} - p^{0.23}\cdot p_{bar}^{1.77}\right)}, & \text{for}\left(\frac{p_{bar}}{p}\right) > 0.54 \\ 1.0, & \text{for}\left(\frac{p_{bar}}{p}\right) \le 0.54 \end{cases}$
4	$\vartheta_{p_{bar}}$	$\begin{cases} \frac{p_{bar}\left(1.53p^{-1.53}\cdot p_{bar}^{0.53} - 1.77p^{-1.77}\cdot p_{bar}^{0.77}\right)}{2\left(\left(\frac{p_{bar}}{p}\right)^{1.53} - \left(\frac{p_{bar}}{p}\right)^{1.77}\right)}, & \text{for}\left(\frac{p_{bar}}{p}\right) > 0.54 \\ 0, & \text{for}\left(\frac{p_{bar}}{p}\right) \le 0.54 \end{cases}$
5	ϑ_χ	0
6	ϑ_T	-0.5
7	ϑ_{ρ_c}	-0.5
8	ϑ_K	-0.5

Table 4. Characteristics of the damaged section of the gas pipeline

Parameter	Symbol	Value	Unit
Inner diameter of the pipeline	D	0.7	m
Outer diameter of the pipeline	D_{outer}	0.72	m
Natural gas pressure upstream of the damage point	p_1	210840	Pa
Natural gas temperature upstream of the damage point	T_1	275.15	K
Natural gas pressure at the damage point of the gas pipeline	p_x	108192	Pa
Natural gas temperature at the damage point of the gas pipeline	T_x	274.0	K
Soil temperature	T_{am}	277.15	K
Pipeline length from the starting point (pressure detection point) to the damage	x	2800	m
Gas volume flow rate reduced to standard conditions	q_{bc}	33.91	m^3/sec

By applying the formulas from Table 1 based on the characteristics of the damaged section of the gas pipeline from Table 4, the impact coefficients for equation (10) were obtained for calculating the uncertainty of the gas pressure at the damage point and presented in Table 5. The uncertainty components included in equation (10) were estimated based on the metrological characteristics of the instruments for measuring gas parameters and the errors of the methods for determining the physical properties of gas given in the relevant standards (see Table 6).

Table 5. Impact coefficients of uncertainty components for equation [10]

№	Impact coefficient of the uncertainty component	
	Symbol	Value
1	ϑ_{p_1}	3.7977
2	$\vartheta_{q_{bc}}$, $\vartheta_{\rho_{bc}}$	-2.7977
3	ϑ_z , ϑ_T , ϑ_x	-1.3988

Table 6. Uncertainty components of equation [10] **for calculating the uncertainty of gas pressure at the at the damage point of the gas pipeline**

№	Uncertainty component		Uncertainty estimation method
	Symbol	Value	
1	u'_{p_1}	0.075%	Based on the main reduced error of the pressure measuring transducer installed upstream of the damage point, δ_p = 0.15%
2	$u'_{q_{bc}}$	0.5%	Based on the main reduced error of the flowmeter installed upstream of the damage point, δ_{qbc} = 1.0%
3	$u'_{\rho_{bc}}$	0.36%	Based on the absolute limit error of gas density measurement under base conditions, $\Delta_{\rho bc}$ = 0.005 kg/m^3
4	u'_z	0.05%	According to the standard [14] or [15]
5	u'_T	0.05%	Based on the absolute limit error of the temperature measuring transducer installed upstream of the damage point, Δ_T = 0.25 °C
6	u'_x	0.09%	Considering the deviation of the actual length from the design length, Δ_{Ldev} = 5 m

Table 7. Impact coefficients of uncertainty components for equation [11]

№	Impact coefficient of the uncertainty component	
	Symbol	Value
1	$\vartheta_{T_{am}}$	0.1949
2	ϑ_{T_1}	0.8085
3	ϑ_x	0.0013
4	$\vartheta_{q_{bc}}$, $\vartheta_{\rho_{bc}}$	-0.0013

Table 8. Uncertainty components of equation [11] **for calculating the uncertainty of gas temperature at the damage point of the gas pipeline**

№	Uncertainty component		Uncertainty estimation method
	Symbol	Value	
1	u'_{T_1}	0.05%	Based on the absolute limit error of the gas temperature measuring transducer installed upstream of the damage point, Δ_T = 0.25 °C
2	$u'_{T_{am}}$	0.18%	Based on reference data [16]
3	$u'_{q_{bc}}$	0.5%	Based on the main reduced error of the flowmeter installed upstream of the damage point, δ_{qbc} = 1.0%
4	$u'_{\rho_{bc}}$	0.36%	Based on the absolute limit error of gas density measurement under base conditions, $\Delta_{\rho bc}$ = 0.005 kg/m^3
5	u'_x	0.09%	Considering the deviation of the actual length from the design length, Δ_{Ldev} = 5 m

Table 9. Impact coefficients of uncertainty components for equation [12]

№	Impact coefficient of the uncertainty component	
	Symbol	Value
1	ϑ_{C_f} , $\vartheta_{F_{hole}}$	1.0
2	ϑ_p	6.5053
3	ϑ_{p_b}	-5.5053
4	ϑ_χ	0
5	ϑ_T , $\vartheta_{\rho_{bc}}$, ϑ_K	-0.5

The relative standard uncertainty u'_p of the natural gas pressure at the damage point of the gas pipeline was calculated by equation (10) using the values of the impact coefficients and uncertainty components presented in Table 5 and Table 6. It is equal to 1.75 %.

By applying the formulas from Table 2 based on the characteristics of the damaged section of the gas pipeline from Table 4, the impact coefficients of the uncertainty components for equation (11) were obtained and presented in Table 7. The uncertainty components were estimated based on the metrological characteristics of the measuring instruments for gas parameters and the errors of the methods for determining the physical properties of gas (see Table 8).

The relative standard uncertainty u'_T of the natural gas temperature at the damage point of the gas pipeline was obtained by equation (11) using the values of the impact coefficients and uncertainty components presented in Table 7 and Table 8. It is equal to 0.054 %.

Table 10. Uncertainty components of equation [12] **for calculating uncertainty of gas flow rate caused by damage to the gas pipeline**

№	Impact coefficient of the uncertainty component		Uncertainty estimation method
	Symbol	Value	
1	u'_{C_f}	0.85%	According to the error of the equation (3)
2	$u'_{F_{hole}}$	0.03%	Based on the limit absolute error of the measuring device for the inner diameter of the pipeline, Δ_{lim}=0.1 mm
3	u'_p	1.75%	Obtained from the gas pressure uncertainty equation at the damage point of the gas pipeline (10)
4	u'_{p_b}	0.02%	By the absolute error of the barometer (0.25 mmHg)
5	u'_T	0.054%	Obtained from the gas temperature uncertainty equation at the damage point of the gas pipeline (11)
6	$u'_{\rho_{bc}}$	0.36%	Based on the absolute limit error of gas density measurement under base conditions, $\Delta_{\rho bc}$ = 0.005 kg/m^3
7	u'_K	0.05%	According to the standard [14] or [15]

By applying the formulas from Table 3 to the considered example (see Table 4), the impact coefficients of the uncertainty components for equation (12) were obtained and presented in Table 9. The values of the uncertainty components included in equation (12) are presented in Table 10.

Based on the values of the impact coefficients from Table 9 and the uncertainty components from Table 10, the relative standard uncertainty of the natural gas flow rate caused by damage to the gas pipeline was obtained by equation (12) - u'_{Qbc}=11.42 %.

When determining the volume of gas lost through damage, the leakage process is divided into three stages:

stage 1 - short-term intensive gas leakage immediately after the gas pipeline rupture;

stage 2 - pseudo-stationary gas leakage through damage at reduced gas pressure and gas entering the damaged section from other gas pipelines;

stage 3 - gas leakage from the damaged section of the gas pipeline after valves closing and disconnecting.

The mathematical models (1) - (3) and analytical dependencies (10) - (12) proposed in this article were used to estimate the uncertainty of the volume of gas lost during stage 2 - pseudo-stationary gas leakage. Taking into account that the gas flow rate through damage during stage 2 is constant, the volume of lost gas can be determined by the formula

$$V_{bc} = Q_{bc} \cdot t \tag{13}$$

where:

- V_{bc} - is the gas volume lost during stage 2, reduced to base conditions; t is the duration of pseudo-stationary gas leakage (stage 2).

The uncertainty of lost natural gas volume calculated by (13) is determined by the equation:

$$u'_{V_{bc}} = \sqrt{{u'_{Q_{bc}}}^2 + {u'_t}^2} \tag{14}$$

The authors considered the case when the valves were manually closed to disconnect the damaged section of the gas pipeline. Therefore, an additional uncertainty component was taken into account, caused by the inaccuracy of estimating the duration of valve closing. Thus, we have established that the uncertainty u'_t equals 1.0%. Then, the combined standard uncertainty of the volume of gas lost through damage because of a complete rupture of the gas pipeline is u'_{Vbc}=11.46 %.

Conclusions

In case of damage to the gas pipeline transport infrastructure, along with the rapid elimination of the damage, it is also important to determine the lost gas volume and estimate the gas volume uncertainty. The authors propose dividing the gas leakage process into three stages and considered the pseudo-stationary gas leakage stage to determine the lost gas volume. During this stage, a mathematical model of stationary gas flow in pipelines can be used to determine the gas parameters at the pipeline damage point.

Based on the analytical solution of this mathematical model, the authors developed an equation for calculating the uncertainty of gas pressure and temperature at the pipeline damage point. It considers the impact of uncertainties in measuring gas pressure and temperature at the point of their registration upstream of the gas pipeline, as well as the distance from the registration point to the damage.

Applying the approaches proposed in ISO 5168:2013 to estimating flow rate uncertainty as a functional dependence on gas parameters and characteristics of the leakage, the authors developed an equation for calculating the uncertainty of gas flow rate through holes in a damaged gas pipeline. The developed equations for the uncertainty of gas pressure and temperature at the pipeline damage point and the uncertainty of gas flow rate through damage make it possible to estimate the uncertainty of the gas volume lost through damage to the gas transportation infrastructure.

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